Homework 2

1. $\lim_{x\to 0} \frac{sin\ x}{5x} = \lim_{x\to 0} [\frac{1}{5} \times \frac{sin\ x}{x}] = \frac{1}{5} \times 1 = \frac{1}{5}$

2. $f(x) = \frac{3x+4}{x^2-16} = \frac{3x+4}{(x+4)(x-4)}$

   $\lim_{x\to 4^-} \frac{3x+4}{(x+4)(x-4)} = -\infty$ $\lim_{x\to 4^+} \frac{3x+4}{(x+4)(x-4)} = \infty$

   Therefore, $x = 4$ is a vertical asymptote.

   $\lim_{x\to -4^-} \frac{3x+4}{(x+4)(x-4)} = -\infty$ $\lim_{x\to -4^+} \frac{3x+4}{(x+4)(x-4)} = \infty$

   Therefore, $x = -4$ is a vertical asymptote.

3. $f'(x) = \lim_{\Delta x\to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} = \lim_{\Delta x\to 0} \frac{\frac{1}{x+\Delta x-1} - \frac{1}{x-1}}{\Delta x} = \lim_{\Delta x\to 0} \frac{(x-1)-(x+\Delta x-1)}{\Delta x(x+\Delta x-1)(x-1)}$

   $= \lim_{\Delta x\to 0} \frac{-\Delta x}{\Delta x(x+\Delta x-1)(x-1)} = \lim_{\Delta x\to 0} \frac{-1}{(x+\Delta x-1)(x-1)} = -\frac{1}{(x-1)^2}$